Calculus AB

1-3

(Day 3)

Evaluating Limits Analytically -Trig Limits

Theorem

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

Theorem

$$\lim_{x\to 0}\frac{1-\cos x}{x}=0$$

For proofs of these theorems, see the text book. They require the use of the Squeeze Theorem.

Evaluate each limit.

1) $\lim_{x\to 0}\frac{\sin(3x)}{3x}$ $= 3\cdot 1 = 3$

For the sin(x)/x theorem to work, we need the same argument in the denominator as in the parentheses. Thus we multiply the top and bottom both by 3.

2) $\lim_{x\to 0}\frac{\tan x}{x}$ $= \frac{\sin x}{x\cos x} = \frac{1}{\cos 0} = 1$

3) $\lim_{x\to \pi}\frac{\sin x}{x}$ $= \frac{0}{\pi} = 0$

4) $\lim_{x\to 0}\frac{1-\cos^2 x}{x}$ $= \frac{(1+\cos x)(1-\cos x)}{x} = 0$

Find $\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$ $= \frac{[(x+\Delta x)^4-1]-[x^4-1]}{\Delta x}$

5) $f(x)=x^4-1$

Pascal's Triangle for Binomial Theorem.

1 1 1
1 2 1
1 3 3 1
1 4 6 4 1

$$\lim_{\Delta x\to 0}\frac{x^4+4x^3\Delta x+6x^2\Delta x^2+4x\Delta x^3+\Delta x^4-x^4+1-1}{\Delta x}$$

$$\lim_{\Delta x\to 0} 4x^3+6x^2\Delta x+4x\Delta x^2+\Delta x^3 = 4x^3$$

Assignment:

Pg. 68

65 - 88